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A first-principles mathematical model integrates the disparate timescales of human learning

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Lifelong learning occurs on timescales ranging from moments to decades. People can lose themselves in a new skill, practice for hours until exhausted, and pursue mastery intermittently over decades. A full understanding of learning requires an account that integrates these timescales. Here, in response to calls for more formal theory in the psychological sciences, we present a parsimonious mathematical model that unifies the nested timescales of learning. Our model recovers well-established patterns of skill acquisition, and explains how these patterns can emerge from short-timescale dynamics of motivation, fatigue, and effort. Conversely, the model explains how patterns in these short-timescale dynamics are shaped by longer-term dynamics of skill selection, mastery, and abandonment. We use this model to explore the theoretical benefits and pitfalls of a variety of training regimes. Our model connects disparate timescales—and the subdisciplines that typically study each timescale in isolation—to offer a unified, multiscale account of skill acquisition.

Learning is a through-line of human life, from children at play to Picasso at his peak. The process of learning occurs on multiple timescales[1–3]. Over the long timecourse of mastering a skill, learning may appear continuous and gradual as one progresses from novice to expert. But this slow process consists of countless shorter, discrete periods of engagement, lasting from mere moments to entire days. An individual may start working but get bored. They may rest. They may eventually master the skill, or give up, or switch focus to another skill entirely. In short, learning occurs on multiple nested timescales, from the moment-to-moment dynamics of motivation and fatigue, to briefer episodes of sustained effort, to the slow crawl toward mastery.

A full understanding of learning requires an account that integrates these timescales[3]. Existing accounts, however, have typically focused on single timescales. Here, we present a minimal mathematical model that unifies the nested timescales of lifelong learning, offering a theoretical account of well-established patterns at both short- and long-timescales. The model captures the relations between the dynamics of motivation, fatigue, and sustained engagement on the shorter timescale of individual periods of work (e.g., minutes or hours), as well as well-established, classic patterns of learning and performance on the longer timescale of mastery (e.g., days or years).

Our goal is to offer a formal explanation of why similar qualitative patterns in skill learning have been found to occur in a wide range of domains, from juggling to chess. We are thus responding to the call for more formal theory building in the psychological sciences[4–6]. Unlike statistical models, which aim to describe empirical data, or cognitive process models, which aim to capture in detail the internal mental processes that give rise to particular psychological outcomes, our model offers a more foundational account of common patterns found across superficially dissimilar domains of learning. This approach to theory building has been influential in fields such as ecology[7] and statistical physics[8]. Formal models such as the one introduced here are opportunities to specify the minimal components that suffice to give rise to qualitative patterns, thus advancing our theoretical understanding of the origin of those patterns. Here, we offer such an account of multiscale lifelong learning.

Below, we briefly review the longer and shorter timescales of learning. We then introduce our formal model, which unifies these timescales.

An often-repeated cliché holds that mastery requires "10,000 h"—that is, two years of constant, daily effort, breaking only to sleep at night[9,10]. While overly simplifying, this saying captures the fact that real mastery requires sustained engagement over long periods[9–13].

The dynamics of learning on this timescale are typically described using so-called "learning curves" or "progress curves"[12,14–16]. These represent the gradual increase in skill over time, where skill is typically measured at the level of an entire session of work or practice: the final score in a game, the best performance in some athletic skill within a training session, mean

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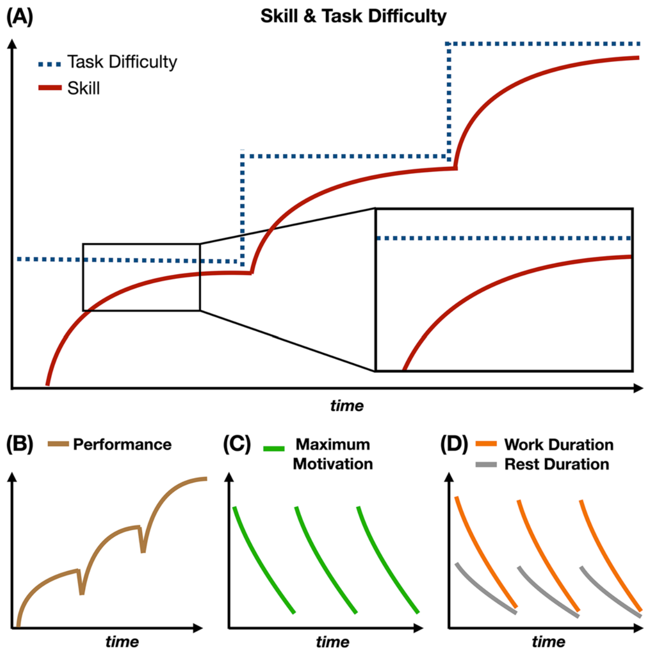


Fig. 1 | Classic patterns in the dynamics of learning, performance, and sustained engagement. Our mathematical model aims to recover these qualitative patterns. **A** Skill development (solid red) typically exhibits diminishing returns on practice as one approaches mastery of a task. Over time, one can switch to more challenging tasks (dashed blue line), thus creating opportunities for continued skill development. Zoomed inset shows a typical learning curve with diminishing returns over time. **B** Performance typically increases with practice but can reach plateaus. When one switches to a more challenging task, there can be a momentary dip in performance before performance outstrips the previous plateau. **C** For people who are motivated by skill development, motivation typically peaks when the task is sufficiently difficult to maximize learning. As the gap shrinks between skill and task difficulty, motivation typically decreases. Switching to a more challenging task can increase motivation. **D** Average work and rest period lengths. When individuals motivated by skill development encounter a task that is sufficiently but not overly challenging, they typically work for long periods (orange), and require comparatively little rest (grey). Over time, as the gap shrinks between skill and task difficulty, they may work for shorter and shorter periods as they get increasingly bored. Note that these dynamics reflect endogenous behavior of the model without external forcings.

typing speed within a testing period, etc. In domains as varied as music, chess, memory, typing, sport, and juggling, progress curves typically show diminishing returns, with the greatest rate of learning when skill is far from mastery, and a gradually diminishing rate of learning as mastery is approached (Fig. 1A, inset)[12,14–17]. Empirical progress curves can be described using power or exponential functions, although there is debate about the best-fitting functional form[18–22].

Indeed, more recent work has found that, at the individual level, learning often improves in distinct stages (Fig. 1A), reflected in plateaus in the learning curve[18,23,24]. These plateaus are periods where the learner has mastered a particular task and has not yet switched their focus to a more challenging one. The learner may eventually pursue a more challenging task, which might result in worse performance at first but ultimately better performance as the new, more challenging task is mastered (Fig. 1B)[23]. In some instances of this general scenario, the simpler "task" is a sub-optimal approach to a problem to which the more difficult "tasks" are better but more challenging solutions. An example of this is when athletes in the sport of high jump start with easier but less effective ways of jumping over the bar (e.g., jumping forwards) but eventually transition to the more challenging but ultimately more effective "Fosbury Flop" (i.e., jumping backwards). The Fosbury Flop was originally derided as an overly difficult approach to the problem of jumping over the bar, and it is indeed more challenging at first than jumping face first, but it is ultimately a more effective solution; indeed, it is now the standard approach at the highest levels of the sport[23]. Thus, on the slow timescale of skill mastery, individuals typically exhibit learning with diminishing returns, and may exhibit repeated plateaus, followed by brief dips in performance and increased learning. This general pattern has been documented in a striking range of domains, from juggling to mental arithmetic, from sports to video games[23].

The slow process of skill mastery, however, is built out of many brief sessions of practice and learning[25,26]. People choose to start working, and then persist in that effort until they give up or require rest—and then, having rested, they may choose to start once more. When people start and stop is often a function of their motivation[27] and fatigue[28] (Exceptions include cases where people are forced to work by some external entity, such as a teacher, coach, or employer). While the development of skill and performance are typically measured at the level of entire work-sessions, people are nevertheless learning within each session. On this fast timescale of moment-to-moment engagement, motivation and fatigue may fluctuate, skills may creep ever upward, and accomplishments may accumulate slowly. Long-term mastery, therefore, emerges from the the dynamics of short-timescale engagement and learning.

A full understanding of life-long learning and sustained engagement requires an understanding of both long and short timescales[3]. The precise timescale on which "long timescale" learning occurs will depend on the particular task and domain. Mastering a new video game, for instance, may require hours or days, while becoming a chess master can take decades. From our perspective, these are both "long timescale" processes, since they involve changes that are orders of magnitude slower than moment-to-moment fluctuations in motivation and fatigue. At present, however, we lack a unifying framework, one that situates known patterns on the short timescale of single work sessions within the context of well-established patterns of long-timescale skill development.

This is a problem because changes on one timescale shape changes on the other[29]. Lifelong learning emerges from the moment-to-moment dynamics of motivation, fatigue, skill development, and performance[30]. The decision to work or rest, meanwhile, is a function of fatigue and motivation, but motivation is shaped by long-timescale changes in skill and in the task being learned[31]. The dynamics of each timescale only make sense in light of the other.

Encouraged by successful models of nested timescales in other biological and social systems[32–35], here we develop a first-principles mathematical model that unifies the nested timescales of learning. By bridging between levels—the long timescale of mastery, and the short timescale of effort and engagement—the model connects phenomena that have traditionally been investigated by distinct research traditions. To be clear, we are not offering a statistical model of empirical data from any particular study of skill acquisition. Moreover, any minimal model of a phenomenon as complex as human lifelong learning will only be able to account for major qualitative patterns. And outcomes that reflect factors that lie outside the model—such as exogenous pressures to work or rest, whether driven by diurnal cycles or workplace supervisors—will necessarily lie outside the model's scope. Rather, our goal is to offer a task-agnostic account that explains, from first-principles, why similar patterns have been found to recur across domains, from chess to juggling. Our contribution is thus theoretical, not empirical.

In what follows, we introduce the formal model. We show that the model reproduces many classic, well-established patterns in skill acquisition, on multiple timescales. This first-principles model thus offers a theoretical account of why similar patterns recur across domains. We then explore the model's behavior in the context of different training regimes (e.g., continuously increasing task difficulty vs. discrete jumps in difficulty) and for individuals who differ in their motivation.

The mathematical model

We present a first-principles model that integrates the multiple timescales of learning, from moment-to-moment engagement to life-long growth and skill acquisition. We take a dynamical systems approach, where our model consists of a system of ordinary differential equations. The dynamical

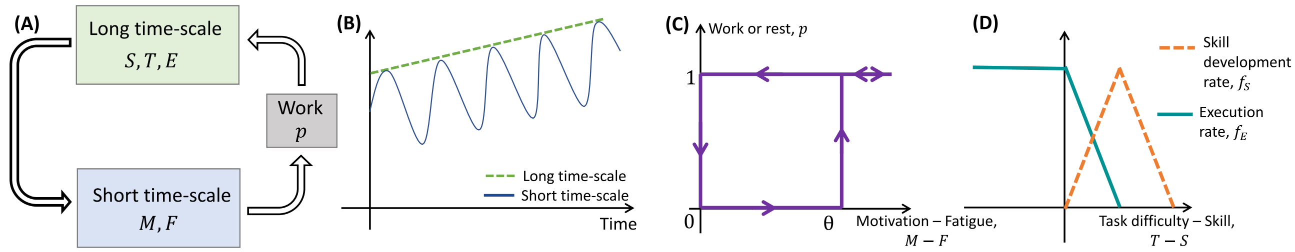


Fig. 2 | Conceptual illustration of the modeling framework and key variables. **A** Short timescale variables (motivation, M, and fatigue, F) determine whether an individual decides to work, p. As an individual works, skill (S) improves as does task execution (E). Task difficulty (T) can change over time or remain constant. **B** Illustration of the distinction between long- and short-timescale variables. Long timescale variables (e.g., skill) tend to change slowly (dashed green line). Short timescale variables (e.g., motivation) change more rapidly, sometimes oscillating (solid blue curve). **C** The decision to work ($p = 1$) or rest ($p = 0$) is a function of the difference between motivation and fatigue ($M - F$). Starting to work requires sufficient motivation relative to fatigue (i.e., $M - F > \theta$). Work stops when fatigue overtakes motivation (i.e., net motivation, $M - F \leq 0$). The additional effort required to start working, θ, induces path-dependence in the decision to work or rest (i.e., hysteresis). **D** Skill S and execution E change more slowly, and their dynamics depend on the relative task difficulty (i.e., $T - S$).

Table 1 | Summary of key model variables and parameters

Variable	Meaning	Range
Long timescale variables		
S	Individual's skill level	$[0, \infty)$
E	Individual's rate of execution on task	$[0, \infty)$
T	Task difficulty	$[0, \infty)$
Short timescale variables		
M	Individual's current motivation	$[0, \infty)$
F	Individual's current fatigue	$[0, \infty)$
p	Whether individual is working (1) or resting (0)	$\{0, 1\}$

Parameters

Parameter	Meaning	Range	Typical value
w	Whether individual is more driven by the accumulation of achievements (0) or skills (1)	[0, 1]	0.5
c_1	Weight of contribution from return to baseline to motivation	$[0, \infty]$	0.5
c_2	Weight of contribution from learning and execution to motivation	$[0, \infty]$	1.2
c_3	Rate of fatigue increase while working	$[0, \infty]$	0.1
c_4	Rate of fatigue decrease while resting	$[0, \infty]$	0.1
x_m, y_m	Shape parameters for learning and execution function f_S, f_E	$[0, \infty]$	1, 0.2
$\theta_{\min}$	Minium activation threshold to switch to working from resting	$[0, \infty]$	0.005
$\epsilon_{\min}$, k	Parameters to determine the threshold for increasing task difficulty (ϵ) in stepwise scenario	$[0, \infty]$	0.005, 0.01

systems approach has been highly effective at providing elegant explanations of complex phenomena in human systems such as motor coordination[36–38], cognitive development[3,24], left-handedness in a minority of the population[39], and the polarization of political parties[40,41]. We divide the description of the dynamical system into two components: three state variables that change on a long timescale, and another three that change on a short timescale. The mathematical model thus contains variables that change on both short and long timescales, with the different timescales connected by the individual's decision to work or rest. Figure 2A illustrates the mathematical model's conceptual framework and the relationships between key variables. See Table 1 for a summary of key model parameters on various time scales.

Long timescale

The three key long-timescale variables are $S(t)$, the skill level of an individual at time t, $T(t)$, the task difficulty, and $E(t)$ (derived from S and T), the rate of execution, or how much work an individual is getting done at time t. Within psychology, rate of execution is sometimes referred to as efficiency[42], though here we use 'rate of execution' to avoid confusion with other uses of the term 'efficiency' in economics and engineering. The variables S, E, and T all take continuous, non-negative values. The distinction between skill and execution captures the fact that people often learn best and perform best at different times[43]. One performs best when executing tasks that are easy, relative to one's skill level, while one develops skills when working on tasks that are more difficult (but not too much more difficult) than one's current skill level[44,45]. We calculate an individual's performance as the product of their current skill and current execution, $S(t)E(t)$, since performance is jointly affected by skill level and rate of execution.

Skill development and execution. In the model, skill improvement is a function of *relative task difficulty*, which is the difference between task difficulty and one's skill level ($T(t) - S(t)$). In other work, this is also referred to as functional task difficulty[44]. Specifically, skill improves when the task is harder than one's current skill level ($T(t) - S(t) > 0$), but only to an extent. If a task is too easy or too difficult, the individual working on it does not develop skill[44,45]. Put mathematically, the dynamics of skill are given by:

$$\frac{dS(t)}{dt} = p(t)f_S(T(t) - S(t)), \tag{1}$$

where $p(t)$ is a boolean variable denoting whether the individual is working at time t. If they are working, $p(t) = 1$, and if resting, $p(t) = 0$. Note that all variables and parameters in this and the following equations are dimensionless. In the Supplemental Information, we show how these dimensionless equations are derived from dimensional ones. The function f_S represents how task difficulty, relative to the current skill level, affects the rate of skill acquisition. For simplicity, we consider this function to take the shape of a triangle, peaking at value x_m (where $x_m > 0$), which we visualize in Fig. 2D. The function can be expressed in the following piecewise form:

$$f_S(x) = y_m \begin{cases} 0 & \text{if } x<0, \\ x/x_m & \text{if } 0 \le x \le x_m, \\ (-x + 2x_m)/x_m & \text{if } x_m < x \le 2x_m, \\ 0 & \text{if } x > 2x_m, \end{cases} \tag{2}$$

This piece-wise function peaks at height y_m, denoting the maximum learning rate. Note that this approach, unlike power-law or exponential learning curves, allows us to model the period of skill acquisition when the initial task is much too difficult for the learner, given their current skill. During this period, learning is laborious and slow, with little skill acquisition occurring even with extended effort. Think of the young child who first encounters chess before they are sufficiently mature to even remember all the rules: repeated encounters with the game will produce little improvement in their ability to play chess. This period is often ignored in empirical work on skill acquisition, because participants and tasks are intentionally matched so the task is approachable. However, in many real-world scenarios, an individual encounters a task that is too difficult to even start—or if starting is possible, then it is initially quite laborious. In our modeling framework, this period corresponds to the right-hand side of the learning function where relative task difficulty is very high.

Second, our approach is agnostic to the exact functional form of the learning curve. We show in the Supplementary Information that the function f_S, which governs the rate of learning as a function of the relative task difficulty, can be modified to generate learning curves that follow, exactly, either an exponential or a power law curve. Here, we choose the simplest functional form that gives diminishing returns on effort: a piecewise linear relationship between relative task difficulty and rate of learning. This allows us to capture the qualitative features of empirical learning curves with fixed tasks, while also allowing us to model more complex scenarios involving variable task difficulty (e.g., when task difficulty is simultaneously changing with time or adapting to skill).

Similarly, the rate of execution, a dimensionless quantity that indicates the proportion of skill level brought to conduct the task, is given by,

$$E(t) = p(t) f_E(T(t) - S(t)), \tag{3}$$

The shape of the function f_E is sketched in Fig. 2D, which is a positive constant when relative task difficulty is negative ($T(t) - S(t) < 0$), and decays and eventually settles to 0 when relative task difficulty increases ($T(t) - S(t) > 0$). The function can be expressed in the following piecewise form:

$$f_E(x) = y_m \begin{cases} 1 & \text{if } x < 0, \\ (x_m - x)/x_m & \text{if } 0 \le x \le x_m, \\ 0 & \text{if } x > x_m. \end{cases} \tag{4}$$

Intuitively, this says that execution is at a maximum when the demands of the task are fully met by one's skill, but execution decreases linearly as the deficit between task difficulty and skill gets larger[44].

An individual's performance on a task reflects both their skill and execution. Performance is highest when they are both highly skilled and executing the task with maximum execution; it is lower for individuals with lower skill, individuals who are struggling to execute the task, or both. We thus measure an individual's performance as the product of their skill and their rate of execution, $S(t)E(t)$.

Task difficulty. Unlike skill ($S(t)$) and execution ($E(t)$), which are solely governed by the internal dynamics of the dynamical system, our third long-timescale variable, task difficulty ($T(t)$), can be treated as an external input variable. As a result, our modeling framework allows us to explore how different ways of increasing task difficulty affect the dynamics of long-timescale learning and short-timescale engagement.

Specifically, we evaluated several different training regimes: task difficulty remains unchanged (constant scenario), increases in a discrete fashion once skill catches up with task difficulty (stepwise scenario), and increases in a continuous fashion (continuous scenario).

Short timescale

On the short timescale, three key variables that govern the individual's working behavior are motivation ($M(t)$), fatigue ($F(t)$), and work ($p(t)$)[46–48]. Motivation refers to one's current willingness or desire to work on a task[49]. For simple tasks that are primarily physical, this may be driven by levels of the neurotransmitter dopamine[50]; for more complex or conceptual tasks, motivation intensity is likely a composite of different psychological and physiological cues. Here, we remain agnostic about the specific physiological underpinning of motivation. We model motivation, $M(t)$, as a continuous variable taking non-negative values.

Fatigue is a sense of cumulative exertion or tiredness[51]. For some simple tasks that are primarily physical, the sense of fatigue may be driven by simple physiological processes, such as the accumulation of cerebral adenosine[50]; for more complex or conceptual tasks, much like for motivation, the sense of fatigue is likely a composite of different psychological and physiological cues. Here we also remain agnostic about the specific physiological underpinning of fatigue. We model fatigue, $F(t)$, as a continuous variable taking non-negative values.

Motivation. We assume the level of motivation is affected by two processes. The first is that, in the absence of other influences, motivation will restore to a baseline[52,53]. In dimensionless equations, this baseline is represented as 1 (see Supplementary Information for details on the nondimensionalization process). The second is that motivation increases when an individual is learning (Eq. (1)) or executing a task (Eq. (3))[54,55]. Mathematically, these dynamics are expressed as

$$\frac{dM(t)}{dt} = c_1(1 - M(t)) + c_2\left[w\frac{dS(t)}{dt} + (1-w)E(t)S(t)\right], \tag{5}$$

where the first term reflects the process of returning to baseline, and the second term captures the increase in motivation due to learning and performance. The parameters c_1 and c_2 are constants weighting the contribution of these two processes. As introduced above, $S(t)$ denotes skill and $E(t)$ denotes execution of an individual at time t. The relative influence of learning and performance on motivation is determined by parameter w, which takes values between 0 (only performance matters) and 1 (only learning matters). Simplifying Eq. (5), we have

$$\frac{dM(t)}{dt} = c_1(1 - M(t)) + c_2 p(t)\left[w f_s(T(t) - S(t)) + (1-w)S(t) f_E(T(t) - S(t))\right]. \tag{6}$$

Fatigue. The level of fatigue is also affected by two processes. The first is that when one is working, the level of fatigue increases[56]. Here, we use the simplest form of this process, a linear increase. The second is that when one is resting, the level of fatigue recovers to baseline, zero. Here, we draw on the literature on recovery (e.g.,ref.[28]), which has found that recovery tends to involve an exponential decay in fatigue. These dynamics of accumulating and decaying fatigue are mathematically integrated into the following conditional function,

$$\frac{dF(t)}{dt} = \begin{cases} c_3 & \text{if } p(t) = 1, \\ -c_4 F(t) & \text{if } p(t) = 0. \end{cases} \tag{7}$$

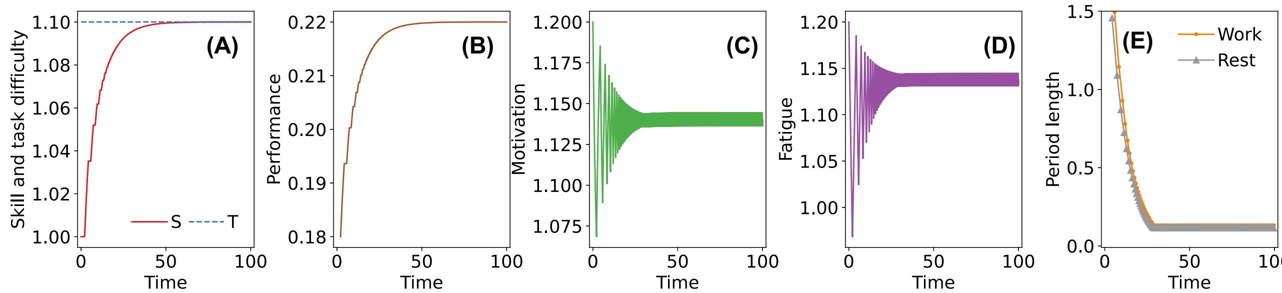


Fig. 3 | Constant task difficulty leads to plateauing skill and performance on the long timescale (A, B), and reduced motivation and working periods on the short timescale (C–E). **A** Skill (red solid line) approaches the task difficulty (blue dashed line), and eventually plateau. **B** Performance of an individual ($S(t)E(t)$), increases over time but eventually plateau. **C, D** Motivation and fatigue oscillate across recurring working sessions, where individual enter a work session when motivation is sufficiently greater than fatigue and exit the work session when motivation drops below fatigue. Peak motivation value of each work session diminishes as one's skill and performance plateau. **E** The durations of an individual's working period and rest period both decline over time, consistent with the decreasing motivation. Note that in the absence of some additional mechanism for sustaining engagement, the individual initiates work but immediately abandons the task, as it is no longer sufficiently motivating to sustain work for more than a moment. In the real world, an individual at this point would likely abandon the task entirely or would require some other mechanism to sustain their engagement, whether endogenous to the individual (e.g., an explicit dedication to persisting despite the lack of reward) or exogenous (e.g., a supervisor or coach who encourages sustained work).

This differential equation formulation is equivalent to a linear increase in time when working, and exponential decay when resting. Constants c_3 and c_4 set the timescales of the changes.

Work. An individual's decision to work or rest depends on their motivation and fatigue[57,58]. Recall that the variable $p(t)$ indicates whether an individual is working ($p(t) = 1$) or resting ($p(t) = 0$). $p(t)$ is determined by the difference between motivation and fatigue, $M(t) - F(t)$. An individual will switch to working from resting when they have sufficient motivation relative to their fatigue—that is, when $M(t) - F(t)$ increases across a critical threshold, $\theta(t)$, which increases with the difference between task difficulty and skill level, while limited to a minimum value, $\theta(t) = \max(T(t) - S(t), \theta_{min})$. The minimum $\theta_{\min}$ reflects the minimum motivational start-up cost of starting to work. Individual stops working when their fatigue is equal to or greater than their motivation—that is, when $M(t) - F(t)$ is smaller than or equal to zero. A sketch of this dynamic is shown in Fig. 2C. In discrete time, the function $p(t)$ can be expressed as

$$p(t + \Delta t) = \begin{cases} 0 & \text{if } p(t) = 0 \text{ and } M(t) - F(t) < \theta, \text{ or } p(t) = 1 \text{ and } M(t) - F(t) \leq 0, \\ 1 & \text{if } p(t) = 0 \text{ and } M(t) - F(t) \geq \theta, \text{ or } p(t) = 1 \text{ and } M(t) - F(t) > 0, \end{cases} \tag{8}$$

where Δt is a small time interval. The activation threshold should be lower for individuals at higher skill levels relative to task difficulty. In other words, it is easier for an individual to start working on a task when they are good at it.

Results

Diminishing return in skill and performance under constant task difficulty

We first analyze a baseline scenario where an individual experiences constant task difficulty over time, illustrated in Fig. 3A (hereafter *constant scenario*). Real-world examples of this constant scenario are ubiquitous, at least within delimited time periods (e.g., lifting a certain weight, playing a card game against a fixed computer opponent). Note that this scenario is analogous to the regime illustrated in the inset of Fig. 1A. We assume, moreover, that the individual is motivated equally by learning and performance; in the last section of the Results, we weaken this assumption and explore the effects of individual differences in motivation.

In this constant scenario, our model recovers the classic result that long-timescale skill development reaches a plateau of both skill and performance, as shown in Fig. 3A, B. The model thus generates the classic empirical pattern of diminishing returns in skill learning when task difficulty is held constant[12,14–16]. In our model, this pattern emerges naturally from short-timescale dynamics of motivation, fatigue, and engagement.

The model also predicts that long-timescale skill acquisition should be accompanied by changes in short-timescale dynamics. As skill improves, both peak motivation and peak fatigue within working sessions decline monotonically (Fig. 3C, D). Moreover, work sessions last longer—that is, engagement is more sustained—when the task difficulty is suitably higher than the skill level (Fig. 3E). Approaching the plateau, one becomes less and less motivated. Consequently, one tends to work for shorter and shorter periods, suggesting an increasing degree of boredom.

The apparently static "plateau" in skill development at the macro-scale is actually the product of an intricate dynamic balance of micro-scale processes. Indeed, in the real-world, the decision to begin working reflects not just the endogenous influences of motivation and fatigue captured by our model, but also exogenous influences such as diurnal cycles (i.e., people are not going to start working while sleeping). Moreover, individuals who are insufficiently motivated by a task will likely require exogenous influences to *persist* in working—a workplace supervisor or a coach, for instance. Such exogenous influences are beyond the scope of our minimal model. In the absence of such external pressures, individuals faced repeatedly with a task that is so unrewarding that it can only sustain their engagement for a moment would likely abandon the task for something more challenging and thus rewarding. The baseline scenario explored in Fig. 3 does not allow for this kind of adjustment; indeed, we assume for simplicity that individuals are forced to repeatedly attempt a task that is no longer rewarding. In subsequent sections, we relax this assumption to explore scenarios where the task is dynamically adjusted or even abandoned entirely for a new, more challenging task.

Sustained skill learning under stepwise increases in task difficulty

In real-world settings, individuals tend to engage in increasingly difficult tasks throughout the long-term learning process. To capture this, we evaluated a scenario where task difficulty increases in discrete jumps whenever an individual's skill catches up with task difficulty, as shown in Fig. 4A (hereafter *stepwise scenario*). Real-world situations corresponding to this situation include strength training with incrementally heavier weights, attempting a new level in a video game, or skipping a grade in school. We set task difficulty to increase by a fixed increment of 0.8 whenever the gap between an individual's skill and the demands of the task (i.e., $T - S$) is smaller than a threshold, ϵ.

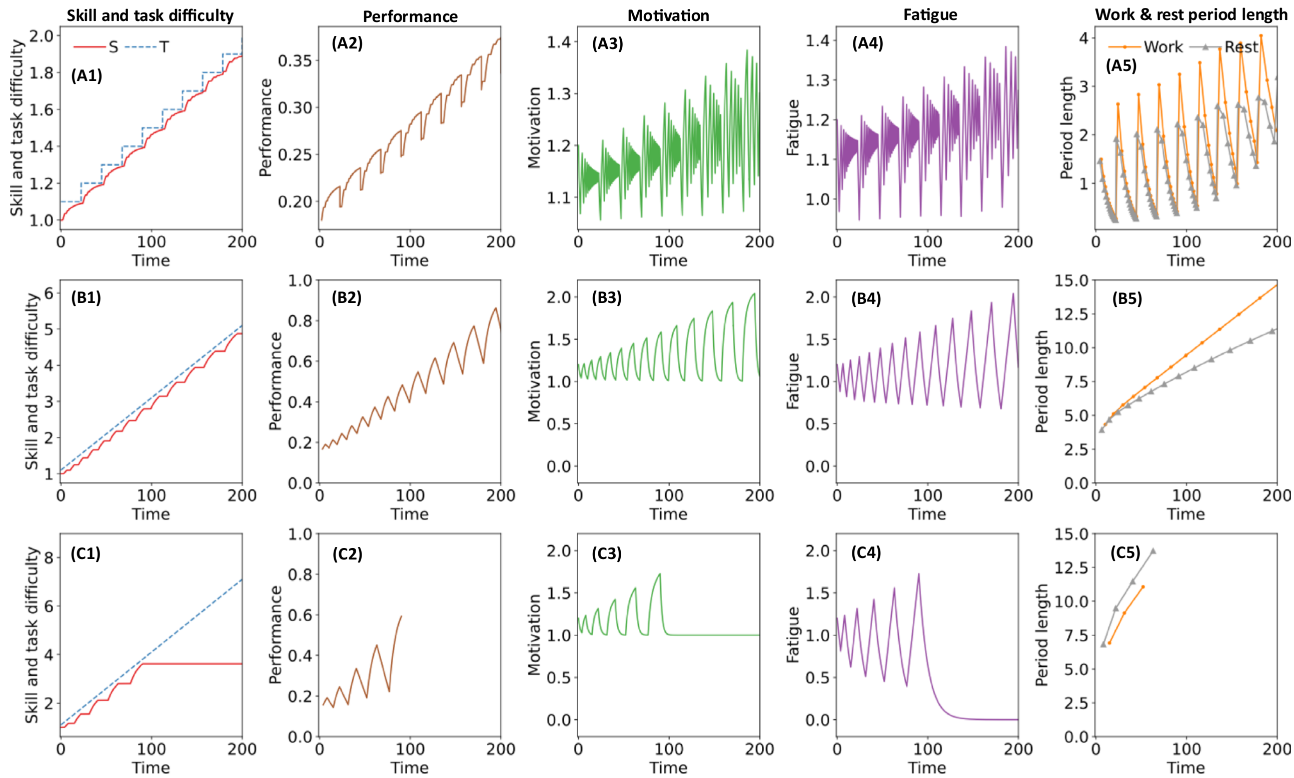


Fig. 4 | Short- and long-timescale dynamics of three distinct learning regimes. **A** Model predictions with stepwise adaptive task difficulty. The model recovers well-established patterns of skill acquisition, performance, and engagement. Note that the qualitative patterns shown here remain robust to variations in the main parameters (see SI for details). **B** Model predictions with linearly increasing task difficulty at a slow pace. The learner adapts to increasing task difficulty by improving skill. **C** Model predictions with linearly increasing task difficulty at a faster pace. Eventually the learner cannot keep up with the fast pace (C1), no longer recovers motivation (C3), and thus stops working entirely (C5). In all panels, parameter $w = 0.5$.

Within each period of constant task difficulty, behavior was similar to that found in the constant scenario, with plateaus in skill and performance as skill approached task difficulty. With repeated, discrete increases in task difficulty, however, both skill and performance surpassed these plateaus (Fig. 4A1, A2).

Our model predicts that when the task difficulty is raised adaptively, one's performance exhibits plateaus, dips, and leaps, as shown in Fig. 4A2. Whenever task difficulty increases after reaching a plateau, performance first dips, then increases to exceed the previous plateau. This qualitative pattern is in line with empirical studies of performance plateaus[22,23].

Similar to the model's predictions in the constant scenario, each episode of task difficulty level starts with a spike in peak motivation, a spike in peak fatigue, and longer duration of working periods. This suggests intense engagement as one first attempts to master a higher task level. Peak motivation, peak fatigue, and working period duration all decline as skill improves and approaches the target task difficulty, essentially recapitulating what we find in the constant scenario (Fig. 4A3–A5). Note that while we assume specific values for the timescale parameters c_1 through c_4 in our simulations, the qualitative findings remain robust to variations in these parameters (see Supplementary Information for details).

Skill learning under continuous increases in task difficulty

To incorporate activities that are not characterized by stepwise change of difficulty, we also examine a scenario where the task becomes increasingly more challenging in a continuous manner (e.g., a gradually increasing speed of running and cycling (Fig. 4B, C).

We observe a qualitatively similar pattern in skill learning and performance compared to that in the stepwise scenario—both scenarios induce sustained increases in skill and performance over the long timescale, with the continuous scenario producing smoother changes.

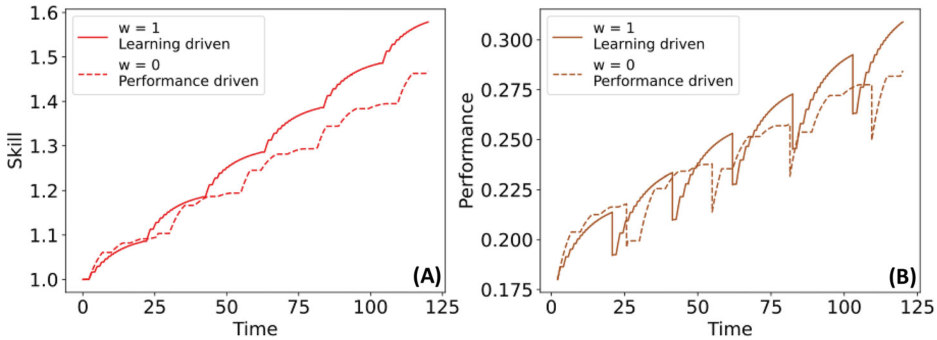
When the rate of increase in task difficulty is not overwhelming, there is a gradual increase in peak motivation, peak fatigue, and the duration of working periods (Fig. 4B3–B5). This indicates increased levels of engagement. This contrasts with what was found in the constant scenario, where peak motivation, peak fatigue, and work period duration all decreased as skill increased (Fig. 3C–E), and is distinct from the periodic changes in the stepwise scenario (Fig. 4A3–A5).

Both the stepwise and continuous scenarios can lead to sustained increase in skill learning and performance, but how far can one push the pace of task difficulty increase? We evaluated a special case in the continuous scenario where we increase the task difficulty at a very high rate (Fig. 4C). In this scenario, individuals give up after failing to keep up with the ever more challenging task (Fig. 4C). Notably, on short timescales, this special case is characterized by a substantial increase in both peak fatigue (Fig. 4C4) and the duration of resting periods (Fig. 4C5). These short-timescale dynamics suggest the task became too demanding, with the individual taking longer recover, without sufficient motivation to compensate for the escalating exertion.

Individual differences: learning-driven individuals overtake performance-driven individuals over the long run

We next analyze the impact of individual differences in motivation on the long-term development of skill and performance. Research in education and social psychology has found that individuals differ in their relationship to challenging learning opportunities: While some individuals are intrinsically motivated to seek out new challenges, others prefer situations where they can demonstrate their existing mastery[59–61]. One study, for instance, found that students who were more intrinsically motivated were more likely to choose a more challenging task, compared to students who were more competent but less intrinsically motivated[60]. We thus sought to simulate individual differences in learners' source of motivation and choice of task.

Fig. 5 | Over the long run, learning-driven individuals achieve higher skill and performance than performance-driven individuals. A Skill learning in individuals motivated by skill ($w = 1$; solid line) or performance ($w = 0$; dashed line). **B** Performance in individuals motivated by skill ($w = 1$; solid line) or performance ($w = 0$; dashed line). When individuals are learning-driven, they are quicker to attempt more difficult tasks, compared to performance-driven individuals. They thus achieve a higher skill and better performance over the long run.



We thus manipulated whether individuals were primarily motivated by learning (i.e., rate of increase of skill) or performance. According to Eq. (5), learning-driven individuals (i.e., high w value) derive proportionally more motivation from their skill development ($dS(t)/dt$), while performance-driven individuals (i.e., low w value) derive proportionally more motivation from their rate of execution ($E(t)$). These two scenarios, therefore, capture two behavioral profiles that have been established in the literature on intrinsic motivation and challenge-seeking.

To capture the finding that intrinsically-motivated individuals are also more likely to seek out more challenging tasks[60], we allowed ϵ—the threshold for increasing task difficulty in our stepwise scenario—to depend on whether individuals are motivated more by learning or performance: $\epsilon = \epsilon_{\min} + kw$, where $\epsilon_{\min} = 0.005$, and the slope $k = 0.01$. For simplicity, in exploring these individual differences, we chose a linear relation between source of motivation (w) and the threshold to seek out a more challenging task (ϵ), although more complicated nonlinear relationships are possible.

As shown in Fig. 5, since performance-driven individuals are slower to attempt more difficult tasks, they suffer fewer dips in performance than learning-driven individuals. This reflects the learning-driven individual's drive to seek out more challenging tasks, even at the cost of short-term dips in performance. The performance of learning-driven individuals, therefore, may repeatedly drop below the performance of performance-driven individuals. This short term deficit, however, is overcome by the learning-driven individuals' improved skill, so that eventually learning-driven individuals perform better than performance-driven individuals, even when the learning-driven individual is experiencing a dip in performance while the performance-driven individual is at their peak.

Discussion

Life-long learning involves non-linear change on multiple timescales. Past research has established a number of classic patterns in these dynamics, patterns that strikingly recur across varied domains[9,22,23,62]. Motivation waxes and wanes from moment to moment[27], skill can develop gradually over days or years[1,2], and performance can rise and dip as new challenges are attempted[23]. Our formal model offers a framework for explaining these known patterns, including how dynamics on one timescale can shape or generate dynamics on another.

Our model offers a parsimonious account of several important phenomena in learning. Learning and performance, for instance, are known to exhibit gradual increases, static plateaus, or short-term dips[22,23,63]. In our model, these long-timescale behaviors emerge from short-timescale fluctuations in motivation, fatigue, and effort. Seemingly static plateaus in long-timescale learning—a recurring pattern in studies of skill acquisition[11,13,14,23]—may thus reflect ongoing dynamic interactions between micro-level processes at short-timescales. Moreover, the peaks and troughs of motivation and fatigue—and thus the amount of time individuals are able to sustain their effort—reflect individual differences in whether one is motivated by intrinsic growth (i.e., skill development) or extrinsic achievement (i.e., performance)[62,64]. The model thus offers a formal account of how individual differences in motivation may relate to long-term patterns of learning. By situating these dynamics within different training regimes—that is, different plans for increasing the task difficulty over time—we investigated how people can achieve life-long learning or fail to maintain their personal growth. Our approach thus offers a general, task-agnostic framework for understanding the multiscale dynamics of learning across varied domains.

Our model does not distinguish between the kind of task (such as between mental and physical skills) and captures the broad-stroke phenomena in motivation, fatigue, and skill learning that transcend specific activities. The model thus offers a formal account of why similar patterns recur across domains. This framework, however, can be modified to account for other aspects of the learning process, including task-specific factors that may play a role only within particular contexts (e.g., extrinsic encouragement from a coach or comrade), or distinct modes or types of learning that may occur during different stages of the learning process (e.g., declarative vs. procedural learning[65]). By formalizing key aspects of lifelong learning, we offer a theoretical foundation on which other more elaborated accounts can build.

In our model, individuals are motivated in part by the gap between their current skill and the difficulty of the task ($T - S$). As a result, individuals do not work at all if the task is too difficult (Fig. 4C1), they experience peak motivation when the task is appropriately difficult relative to their current skill, and motivation decreases as task becomes relatively boring. This period of maximal motivation, moreover, is accompanied by longer periods of sustained effort—despite high levels of accumulated fatigue. In other words, when the task's challenge is aligned with the individual's skill—not too difficult (Fig. 4C1) but not too easy (Fig. 3)—then individuals experience high motivation, the capacity to work for extended periods of time, and the capacity to sustain their efforts in the face of high fatigue (Fig. 4, panels B3–B5).

These emergent attributes of our simulated work are hallmarks of deep, sustained engagement. They resemble features of the phenomenon of "flow," a state of mind that typically occurs when a task is sufficiently difficult—not too difficult to make progress but also not too easy to offer a challenge[62]. Flow experiences consist of a cluster of features, including heightened motivation and an ability to work for long periods despite fatigue[62,64]. This pattern shares striking similarities with the emergent dynamics of work, rest, and motivation in our model, particularly when the task difficulty was neither too challenging nor too easy (e.g., Fig. 4A, B). This raises the possibility—admittedly speculative—that the minimal mechanisms that are formalized in our model may partially explain why flow states consist of the particular cluster of features with which they are associated[62].

While episodes of deep engagement, such as flow states, have traditionally been discussed as a phenomenon that emerges and disappears on short timescales—emerging during a single session of work, or reflecting the fit between current activities and goals[66]—our model offers a framework for situating the short-timescale dynamics of deep, sustained engagement within the long-timescale dynamics of skill development, performance, and task selection. When task difficulty increases in discrete jumps, for instance, individuals experience brief periods of deep engagement, which disappear as their skill catches up to the demands of the task, at which point the task is no longer sufficiently challenging (Fig. 4A). When the difficulty of the task is increased continuously, however, deep engagement can be sustained and even increased over time (Fig. 4B). While our minimal model does not speak to the subjective, phenomenological features of deep, sustained engagement, it offers a parsimonious account of the coupled dynamics of motivation, work, and fatigue on both short- and long-timescales.

The emergence of deep, sustained engagement in our model depends on the source of an individual's motivation. When individuals are intrinsically motivated by learning (i.e., w close to 1), they sacrifice short-term gains in performance for the opportunity to improve their skill (Fig. 5). This leads to frequent dips in performance. Over time, however, the benefits of long-term cumulative learning outweigh the costs of these temporary performance dips induced at the onset of new skill acquisition. This is a phenomenon that has been described empirically in skill acquisition ranging from the discovery of the Fosbury Flop technique in the sport of high jump (initial adoption of the Fosbury Flop can lead to a drop in performance before the technique is mastered), to the development of prodigious memory for a span of random digits[23]. Since performance dips are followed by periods of increased challenge and deep engagement, our model captures the notion of the so-called autotelic personality type, characterized by a disposition to actively seek challenges and deep engagement[67]. The long-timescale dynamics of learning thus depend on the moment-to-moment dynamics of motivation and task selection.

Maximizing long-timescale learning, however, requires a careful balance. For individuals to experience sustained, long-term learning, they must engage in challenges above their current skill level, but only to a degree. If individuals choose to maximize their current performance rather than seek out new challenges (such as the performance-driven individuals in Fig. 5), then their long-term learning will progress more slowly. But if task difficulty is increased too quickly, then learning cannot keep up and eventually an individual will cease to learn, perform, and work (Fig. 4C). When the increase in learning is carefully calibrated to an individual's ability (Fig. 4A, B), then learning and peak motivation can both grow over long periods[68–70].

The decision to attack entirely new tasks, or the discovery of a more difficult but potentially more effective approach to an existing task, can produce learning curves that consist of piece-wise power laws[22]. Here, our model reproduces this long-timescale dynamic when the task difficulty is increased in discrete jumps (Fig. 4A), with each period of decreasing returns followed by a period of renewed rapid growth. In our model, however, each period of decreasing returns actually follows an *exponential* rather than power-law learning curve, in line with work showing that individual learning curves follow an exponential form[14,20,71]. As we show analytically in the Supplementary Information, this exponential decay in the rate of learning emerges from our assumption that the rate of learning depends only on relative task difficulty (i.e., the difference between task difficulty and skill, $T - S$). In order for learning to follow a power law, the rate of learning must depend not only on the amount of current challenge but also on the passage of time itself (e.g., decaying as a function of the number of practice trials). The difference between exponential and power laws of practice, therefore, reduces to whether the classic phenomenon of diminishing returns depends only on changes in skill (i.e., diminishing as skill approaches saturation), or on both skill and time (e.g., also diminishing with each attempt[20]). We speculate, therefore, that the functional form of an individual's learning curve may depend on the individual and the task. In cases where the rate of learning decays with time (e.g., with an individual who just tries less over time), then learning should follow a power law; in cases where the rate of learning depends only on the current challenge, then learning should follow an exponential curve.

In the following paragraphs, we briefly summarize our work and highlight our contributions. Overall, skill mastery is the outcome of processes that operate on widely varying timescales. Motivation and fatigue can change rapidly in response to context and activity, decisions to work or rest are based on fluctuations in motivation and fatigue, while long-term improvements in skill and performance reflect days or years of work.

Our work offers two contributions. First, our model offers a formal mathematical framework to study the interaction of these timescales, and to understand the multiscale dynamics of learning as an integrated dynamical system. The stability of long-term dynamics (e.g., skill plateaus), for instance, emerges from dynamic and intricate interactions on shorter timescales, and the functional form of long-timescale learning curve depends on the instantaneous rate of learning. The influence goes in the other direction, too—different long-timescale regimes for increasing task difficulty, for instance, produce different short-timescale dynamics of motivation and fatigue. While common sense suggests that there must be interactions between processes on these different timescales, our model spells out some of those potential interactions.

Second, our work brings into conversation different research areas that are typically investigated in isolation. Research on life-long mastery is seldom connected to the small improvements possible during a session of practice. The mastery achieved by virtuosos such as the artist Picasso or the athlete Muhammad Ali is treated separately from the modest achievements of regular people. Nevertheless, learning occurs on all timescales, and is common to all humans. This paper is an attempt to bridge these timescales.

The model has a number of limitations. First, since our goal was to capture domain-general patterns in skill acquisition, we have deliberately ignored task-specific constraints on skill acquisition. While learning chess and learning to juggle may share high-level, qualitative patterns, they differ of course in countless ways. Future work should incorporate task-specific considerations—such as limitations of the body, or task-specific strategies—to develop formal accounts of learning within specific domains. Second, our model does not currently account for interactions between different skills and activities. Many contemporary workplaces, for instance, require individuals to adapt to job requirements that change over time, such that workers must replace old skills with new ones, or even develop multiple new skills in parallel. Future work should explore the ways in which acquiring one skill may help, or even hinder, the acquisition of another, and how the pursuit of multiple skills in parallel can be synergistic or detrimental.

As stated in the introduction, our mathematical and computational model answers the call for theory building in psychological science[4] by offering a unified framework for theorizing about the multiple timescales on which human learning unfolds. This is not a statistical model of a particular dataset. Instead, we offer a principled, formal account of a complex phenomenon. In so doing, we hope our work encourages scientists working in separate disciplines to start a collective conversation about the complex, multiscale dynamics of skill learning.

Methods

All methodological details necessary to replicate the model are described comprehensively in the section, "The mathematical model". Additional details on mathematical derivations are included in Supplementary Information.

Code availability

The code used in the simulation, as well as data generated from the simulations, are available as supplementary files.

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Author contributions
M.L. led the conceptualization of the study, V.C.Y. led the simulation of the mathematical model, and all authors contributed to the development of the model and wrote the paper.

Competing interests
The authors declare no competing interests.

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